Problem 43

Two points in a plane have polar coordinates $P_1(2.500 \text{ m}, \pi/6)$ and $P_2(3.800 \text{ m}, 2\pi/3)$. Determine their Cartesian coordinates and the distance between them in the Cartesian coordinate system. Round the distance to a nearest centimeter.

Solution

Find the corresponding Cartesian coordinates to the given polar coordinates.

$$\begin{cases} x_1 = r_1 \cos \theta_1 \\ \\ y_1 = r_1 \sin \theta_1 \end{cases} \qquad\qquad \begin{cases} x_2 = r_2 \cos \theta_2 \\ \\ y_2 = r_2 \sin \theta_2 \end{cases}$$

$$\begin{cases} x_1 = (2.500 \text{ m}) \cos \dfrac{\pi}{6} \\ \\ y_1 = (2.500 \text{ m}) \sin \dfrac{\pi}{6} \end{cases} \qquad\qquad \begin{cases} x_2 = (3.800 \text{ m}) \cos \dfrac{2\pi}{3} \\ \\ y_2 = (3.800 \text{ m}) \sin \dfrac{2\pi}{3} \end{cases}$$

$$\begin{cases} x_1 \approx 2.165 \text{ m} \\ \\ y_1 = 1.250 \text{ m} \end{cases} \qquad\qquad \begin{cases} x_2 = -1.900 \text{ m} \\ \\ y_2 \approx 3.291 \text{ m} \end{cases}$$

Use the distance formula to get the distance between the points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1.900 \text{ m} - 2.165 \text{ m})^2 + (3.291 \text{ m} - 1.250 \text{ m})^2}$$

$$\approx 4.55 \text{ m}$$